Intermediate Value Theorem

The *intermediate value theorem* states that, if a function f is continuous on a **closed** interval $[a, b]$ (that is, an interval that includes both endpoints) and $f(a) \neq f(b)$, then for every $N \in (a, b)$, there exists a $c \in (a, b)$ with $f(c) = N$.

Intermediate Value Theorem

The *intermediate value theorem* states that, if a function f is continuous on a **closed** interval $[a, b]$ (that is, an interval that includes both endpoints) and $f(a) \neq f(b)$, then for every $N \in (a, b)$, there exists a $c \in (a, b)$ with $f(c) = N$.

Note that the theorem does not apply if f is not continuous at every point of $[a, b]$.

Intermediate Value Theorem

The *intermediate value theorem* states that, if a function f is continuous on a **closed** interval $[a, b]$ (that is, an interval that includes both endpoints) and $f(a) \neq f(b)$, then for every $N \in (a, b)$, there exists a $c \in (a, b)$ with $f(c) = N$.

Note that the theorem does not apply if f is not continuous at every point of $[a, b]$.

The theorem also does not apply if $f(a) = f(b)$, that is, if the function value is the same at both endpoints.

IVT

The intermediate value theorem can be used to prove that an equation has a **root**, that is, a point where

$$f(c) = 0, \quad c \in [a, b]$$

IVT

The intermediate value theorem can be used to prove that an equation has a **root**, that is, a point where

$$f(c) = 0, \quad c \in [a, b]$$

If $f(a) < 0$ and $f(b) > 0$, the intermediate value theorem guarantees that f assumes every value between $f(a)$ and $f(b)$ somewhere on the open interval (a, b).

IVT

The intermediate value theorem can be used to prove that an equation has a **root**, that is, a point where

$$f(c) = 0, \quad c \in [a, b]$$

If $f(a) < 0$ and $f(b) > 0$, the intermediate value theorem guarantees that f assumes every value between $f(a)$ and $f(b)$ somewhere on the open interval (a, b).

In this case, since $f(a) < 0 < f(b)$, the theorem guarantees the existince of a $c \in [a, b]$ with $f(c) = 0$.

Example 1

Prove that the equation

$$4 - x^2 = 0$$

has a root in the interval $(1, 3)$

Example 1

Prove that the equation

$$4 - x^2 = 0$$

has a root in the interval $(1, 3)$

Note that $f(x) = 4 - x^2$ is continuous everywhere, so it is also continuous on the closed interval $[1, 3]$

Example 1

Prove that the equation

$$4 - x^2 = 0$$

has a root in the interval $(1, 3)$

Note that $f(x) = 4 - x^2$ is continuous everywhere, so it is also continuous on the closed interval $[1, 3]$

Now note that $f(1) = 4 - 1^2 = 3 > 0$ and $f(3) = 4 - 3^2 = -5 < 0$, so by the intermediate value theorem f has a root in $(1, 3)$.

Example 2

Prove that the equation

$$1 - x + \sqrt{2x} = 0$$

has a root in the interval $(1, 8)$

Example 2

Prove that the equation

$$1 - x + \sqrt{2x} = 0$$

has a root in the interval $(1, 8)$

Note that $f(x) = 1 - x + \sqrt{2x}$ is continuous on $[0, \infty)$, so it is also continuous on the closed interval $[1, 8]$

Example 2

Prove that the equation

$$1 - x + \sqrt{2x} = 0$$

has a root in the interval $(1, 8)$

Note that $f(x) = 1 - x + \sqrt{2x}$ is continuous on $[0, \infty)$, so it is also continuous on the closed interval $[1, 8]$

Now note that $f(1) = 1 - 1 + sqrt2 > 0$ and $f(8) = 1 - 8 + \sqrt{16} < 0$, so f has a root in $(1, 8)$.

Example 2

Let

$$f(x) = \frac{3x+2}{5x-1}$$

Example 2

Let

$$f(x) = \frac{3x+2}{5x-1}$$

As $x \to \infty$, $f(x) \to \frac{3}{5}$ from above.

Example 2

Let

$$f(x) = \frac{3x + 2}{5x - 1}$$

As $x \to \infty$, $f(x) \to \frac{3}{5}$ from above.

As $x \to -\infty$, $f(x) \to \frac{3}{5}$ from below.

Example 3

Let

$$f(x) = 2 + e^x$$

Example 3

Let

$$f(x) = 2 + e^x$$

As $x \to -\infty$, $f(x) \to 2$ from above.

Question 1

Find the horizontal asymptote(s) of

$$f(x) = 2 + \frac{1}{x^2}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = -2$
5. $y = -1$
6. f has no horizontal asymptote

Question 1

Find the horizontal asymptote(s) of

$$f(x) = 2 + \frac{1}{x^2}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = -2$
5. $y = -1$
6. f has no horizontal asymptote

3. $y = 2$.

Question 2

Find the horizontal asymptote(s) of

$$f(x) = 3 + \frac{1}{x^2 - 1}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = 3$
5. $y = -3$
6. f has no horizontal asymptote

Question 2

Find the horizontal asymptote(s) of

$$f(x) = 3 + \frac{1}{x^2 - 1}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = 3$
5. $y = -3$
6. f has no horizontal asymptote

4. $y = 3$.

Question 3

Find the horizontal asymptote(s) of

$$f(x) = \frac{2x^5 + 7x^3 - 15x + 10}{x^5 - 8x^4 + 2x^2 + 3}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = 3$
5. $y = -3$
6. f has no horizontal asymptote

Question 3

Find the horizontal asymptote(s) of

$$f(x) = \frac{2x^5 + 7x^3 - 15x + 10}{x^5 - 8x^4 + 2x^2 + 3}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = 3$
5. $y = -3$
6. f has no horizontal asymptote

3. $y = 2$.