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means that $f(x)$ can be made as large negative as we want by taking x sufficiently large.

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We say that the line $y = L$ is a **horizontal asymptote** if either

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The graph may approach the line $y = L$ from above or below.

Example 1

Let

$$f(x) = 1 + \frac{1}{x}$$

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Example 3

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As $x \to -\infty$, $f(x) \to 2$ from above.

Question 1

Find the horizontal asymptote(s) of

$$f(x) = 2 + \frac{1}{x^2}$$

1. $y = 0$
2. $y = 1$
3. $y = 2$
4. $y = -2$
5. $y = -1$
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Question 2

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$$f(x) = 3 + \frac{1}{x^2 - 1}$$

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Question 3

Find the horizontal asymptote(s) of

$$f(x) = \frac{2x^5 + 7x^3 - 15x + 10}{x^5 - 8x^4 + 2x^2 + 3}$$

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