MA125 Epsilon-Delta examples

1)
Suppose $f(x) = 5x - 2$. Find $\delta > 0$ such that

$$|f(x) - 3| < \frac{1}{4} \text{ whenever } |x - 1| < \delta$$

Solution:

$$|f(x) - 3| = |5x - 2 - 3| = |5x - 5| < \frac{1}{4}$$

This is the same as saying

$$-\frac{1}{4} < 5x - 5 < \frac{1}{4}$$

Dividing all terms by 5 gives

$$-\frac{1}{20} < x - 1 < \frac{1}{20}$$

which is equivalent to

$$|x - 1| < \frac{1}{20} = \delta$$

2) Suppose $f(x) = 5x - 2$. Prove that

$$\lim_{x \to -1} f(x) = -7$$

Proof: Let $\epsilon > 0$ be given. We need to find a $\delta > 0$ such that

$$|f(x) - L| = |5x - 2 + 7| = |5x + 5| < \epsilon$$

whenever

$$|x - a| = |x + 1| < \delta$$

First rewrite

$$|5x + 5| < \epsilon$$

as

$$-\epsilon < 5x + 5 < \epsilon$$

Dividing by 5 gives

$$-\frac{\epsilon}{5} < x + 1 < \frac{\epsilon}{5}$$

which can be written as

$$|x + 1| < \frac{\epsilon}{5} = \delta$$

so given ϵ, let $\delta = \epsilon/5$.

3) Suppose $f(x) = 3x$. Prove that

$$\lim_{x \to 0} f(x) = 0$$

Proof: Let $\epsilon > 0$ be given. We need to find a $\delta > 0$ such that

$$|f(x) - L| = |3x - 0| = |3x| < \epsilon$$

whenever

$$|x - a| = |x| < \delta$$

First rewrite

$$|3x| < \epsilon$$

as

$$-\epsilon < 3x < \epsilon$$

Dividing by 3 gives

$$-\frac{\epsilon}{3} < x < \frac{\epsilon}{3}$$

which can be written as

$$|x| < \frac{\epsilon}{3} = \delta$$

so given ϵ, let $\delta = \epsilon/3$.

4) Suppose

$$f(x) = \frac{1}{4}x + 2$$

Prove that

$$\lim_{x \to 1} f(x) = \frac{9}{4}$$

Proof: Let $\epsilon > 0$ be given. We need to find a $\delta > 0$ such that

$$|f(x) - L| = \left| \frac{x}{4} + 2 - \frac{9}{4} \right| = \left| \frac{x-1}{4} \right| < \epsilon$$

whenever

$$|x - a| = |x - 1| < \delta$$

First rewrite

$$\left| \frac{x-1}{4} \right| < \epsilon$$

as

$$-\epsilon < \frac{x-1}{4} < \epsilon$$

Multiplying by 4 gives

$$-4\epsilon < x - 1 < 4\epsilon$$

which can be written as

$$|x - 1| < 4\epsilon = \delta$$

so given ϵ, let $\delta = 4\epsilon$.

5) Suppose

$$f(x) = \pi x + 1$$

Prove that

$$\lim_{x \to 2} f(x) = 1 + 2\pi$$

Proof: Let $\epsilon > 0$ be given. We need to find a $\delta > 0$ such that

$$|f(x) - L| = |\pi x + 1 - (2\pi + 1)| = |\pi(x - 2)| < \epsilon$$

whenever

$$|x - a| = |x - 2| < \delta$$

First rewrite

$$|\pi(x - 2)| < \epsilon$$

as

$$-\epsilon < \pi(x - 2) < \epsilon$$

Dividing by π gives

$$-\frac{\epsilon}{\pi} < x - 2 < \frac{\epsilon}{\pi}$$

which can be written as

$$|x - 2| < \frac{\epsilon}{\pi} = \delta$$

so given ϵ, let $\delta = \epsilon/\pi$.