More Epsilon Delta Examples

Gene Quinn

Epsilon-Delta

Suppose $f(x) = 7x + 4$. Find $\delta > 0$ such that

$$|f(x) - 11| < 1 \text{ whenever } |x - 1| < \delta$$

Epsilon-Delta

Suppose $f(x) = 7x + 4$. Find $\delta > 0$ such that

$$|f(x) - 11| < 1 \text{ whenever } |x - 1| < \delta$$

Solution: We need to find a $\delta > 0$ such that

$$|f(x) - 11| = |7x + 4 - 11| = |7x - 7| < 1$$

whenever

$$|x - 1| < \delta$$

Epsilon-Delta

We want to manuever the expression

$$|7x - 7| < 1$$

into an equivalent expression that looks like

$$|x - 1| < something$$

Epsilon-Delta

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$$|7x - 7| < 1$$

into an equivalent expression that looks like

$$|x - 1| < something$$

something will be the value we assign delta.

Epsilon-Delta

First note that

$$|7x - 7| < 1$$

means the same thing as

$$-1 < 7x - 7 < 1$$

Epsilon-Delta

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$$|7x - 7| < 1$$

means the same thing as

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Dividing all expressions by 7 gives

$$-\frac{1}{7} < x - 1 < \frac{1}{7}$$

Epsilon-Delta

Now convert back to absolute values,

$$-\frac{1}{7} < x - 1 < \frac{1}{7}$$

means the same thing as

$$|x - 1| < \frac{1}{7}$$

Epsilon-Delta

Now convert back to absolute values,

$$-\frac{1}{7} < x - 1 < \frac{1}{7}$$

means the same thing as

$$|x - 1| < \frac{1}{7}$$

So, choose

$$\delta = \frac{1}{7}$$

Epsilon-Delta

Does this actually work?

Suppose

$$|x-1| < \delta = \frac{1}{7}$$

Epsilon-Delta

Does this actually work?

Suppose

$$|x - 1| < \delta = \frac{1}{7}$$

Note that

$$|x - 1| < \frac{1}{7}$$

means the same thing as

$$-\frac{1}{7} < x - 1 < \frac{1}{7}$$

Epsilon-Delta

Multiply all expressions in

$$-\frac{1}{7} < x - 1 < \frac{1}{7}$$

by 7 to get

$$-1 < 7x - 7 < 1$$

Epsilon-Delta

Multiply all expressions in

$$-\frac{1}{7} < x - 1 < \frac{1}{7}$$

by 7 to get

$$-1 < 7x - 7 < 1$$

Now write $7x - 7$ as $7x + 4 - 11$,

$$|7x + 4 - 11 < 1$$

and this is the same as

$$|f(x) - 11| < 1$$

which is what we wanted.

Epsilon-Delta

Suppose $f(x) = 7x + 4$. Prove that

$$\lim_{x \to 1} f(x) = 11$$

Epsilon-Delta

Suppose $f(x) = 7x + 4$. Prove that

$$\lim_{x \to 1} f(x) = 11$$

Proof: Suppose $\epsilon > 0$ is given. We need to find a $\delta > 0$ such that

$$|f(x) - L| = |7x + 4 - 11| = |7x - 7| < \epsilon$$

whenever

$$|x - a| = |x - 1| < \delta$$

Epsilon-Delta

We want to manuever the expression

$$|7x - 7| < \epsilon$$

into an equivalent expression that looks like

$$|x - 1| < something$$

Epsilon-Delta

We want to manuever the expression

$$|7x - 7| < \epsilon$$

into an equivalent expression that looks like

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something will be the value we assign delta.

Epsilon-Delta

First note that

$$|7x - 7| < \epsilon$$

means the same thing as

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Epsilon-Delta

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means the same thing as

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Dividing all expressions by 7 gives

$$-\frac{\epsilon}{7} < x - 1 < \frac{\epsilon}{7}$$

Epsilon-Delta

Now convert back to absolute values,

$$-\frac{\epsilon}{7} < x - 1 < \frac{\epsilon}{7}$$

means the same thing as

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So, choose

$$\delta = \frac{\epsilon}{7}$$

Epsilon-Delta

Does this actually work?

Suppose

$$|x - 1| < \delta = \frac{\epsilon}{7}$$

Epsilon-Delta

Does this actually work?

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$$|x-1| < \delta = \frac{\epsilon}{7}$$

Note that

$$|x-1| < \frac{\epsilon}{7}$$

means the same thing as

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Epsilon-Delta

Multiply all expressions in

$$-\frac{\epsilon}{7} < x - 1 < \frac{\epsilon}{7}$$

by 7 to get

$$-\epsilon < 7x - 7 < \epsilon$$

Epsilon-Delta

Multiply all expressions in

$$-\frac{\epsilon}{7} < x - 1 < \frac{\epsilon}{7}$$

by 7 to get

$$-\epsilon < 7x - 7 < \epsilon$$

Now write $7x - 7$ as $7x + 4 - 11$,

$$|7x + 4 - 11 < \epsilon$$

and this is the same as

$$|f(x) - L| < \epsilon$$

which is what we wanted.