Epsilon Delta

Gene Quinn

Epsilon-Delta

The cast:

Epsilon D. "Ed" Criterion (the hero)

Just your average Mathematician

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Thomas Biltwright (A Brilliant Civil Engineer)

Designer of the bridge over the Gibraltar Straits

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Thomas Biltwright (A Brilliant Civil Engineer)

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Dari Clay (An evil genius)

Bent on destroying the world by showing that everything we think we know about Mathematics is wrong.

Epsilon-Delta

The opening scene: The function room at Ed's house. Ed is relaxing by meditating on transcendentals.

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Ed: Why would it fall down? Didn't you use calculus to design it?

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Tom: My bridge is scheduled to open tomorrow and it's going to fall down.

Ed: Why would it fall down? Didn't you use calculus to design it?

Tom: Of course I did, but I just discovered that calculus is *wrong*.

Epsilon-Delta

Ed: What do you mean *calculus is wrong*? That's absurd.

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Ed: I'm skeptical. Tell me more about this diabolical function. (hands Tom a marker) Here, just write on the wall, that's what a function room is for.

Epsilon-Delta

Tom: It's pure evil. She took the innocent identity function...(writes on the wall, hesitantly at first)

$$f(x) = x$$

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$$f(x) = x$$

Ed: What harm could possibly come from that?

Tom: Not much, unless you mutate it so it maps the *irrationals* to one:

$$f(x) = \begin{cases} x & \text{if} \quad x \quad \text{is rational} \\ 1 & \text{if} \quad x \quad \text{is irrational} \end{cases}$$

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Ed: Very clever. It's a perfectly good function - passes the vertical line test.

Epsilon-Delta

Tom: You mean a perfectly evil function. Think about the limit as $x \to 0$. According to the definition, the limit is zero if I can make $f(x)$ as close as I want to zero by taking x sufficiently close to zero.

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Tom: You see, if I stick to rational values for x, I can make $f(x)$ as close as I like to zero by taking x sufficiently close to zero. But if I take x irrational, the function is always 1, no matter how close to zero I am.

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Tom: You see, if I stick to rational values for x, I can make $f(x)$ as close as I like to zero by taking x sufficiently close to zero. But if I take x irrational, the function is always 1, no matter how close to zero I am.

Ed: Well, I think the spirit of the definition is that $f(x)$ should be close to zero for *all* of the $x's$ within a certain distance of zero.

Epsilon-Delta

Tom: That may be, but it's not what it *says*. The bridge opens in less than 24 hours. What are we going to do?

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Ed: We've got to come up with a stronger definition - one that can eliminate Dari's evil creation.

Tom: Yes, but how?

Ed: We can start by saying $f(x)$ has to be close to L *whenever* x is sufficiently close to a, something like

$$|f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta$$

That takes care of the loophole Dari exploited.

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Tom: (with a pained look) *Absolute values?* Why do we have to use absolute values?

Epsilon-Delta

Ed: It's the usual way to measure distance on a line. Hey, we're trying to save the world here - just think of them as double inequalities:

$|F(x) - L| < \epsilon$ means the same thing as $-\epsilon < F(x) - L < \epsilon$

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Ed: Oh, that's easy. I get ϵ from you and I figure out δ based on f, ϵ and a.

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Tom: No, I meant the *values* of ϵ and δ.

Ed: Oh, that's easy. I get ϵ from you and I figure out δ based on f, ϵ and a.

Tom: You lost me.

Epsilon-Delta

Ed: OK, lets start with something reasonable like

$$\lim_{x \to 2} 3x + 1 = 7$$

Now I need a value for ϵ from you. Once I have it, I use it to figure out a value of δ so that:

$3x + 1$ is within ϵ of 7

whenever

x is within δ of 2, or, more precisely,

$$|3x + 1 - 7| < \epsilon \quad \text{whenever} \quad 0 < |x - 2| < \delta$$

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Tom: How about $\epsilon = 1$? Will that work?

Epsilon-Delta

Ed: Well, the idea is that I have to be able to come up with a δ that makes it work. Let's see how I would do that given $\epsilon = 1$. We want to find a δ such that

$$|3x + 1 - 7| = |3x - 6| < 1 \quad \text{whenever} \quad 0 < |x - 2| < \delta$$

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Tom: OK, where would you start?

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$$|3x + 1 - 7| = |3x - 6| < 1 \quad \text{whenever} \quad 0 < |x - 2| < \delta$$

Tom: OK, where would you start?

Ed: Start with the expression $|3x - 6| < 1$ and try to manuever it into an equivalent expression that looks like $|x - 2| < something$. That something will be what we call delta.

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Tom: OK, where would you start?

Ed: Start with the expression $|3x - 6| < 1$ and try to manuever it into an equivalent expression that looks like $|x - 2| < something$. That something will be what we call delta.

You're going to like the first step - we get rid of the absolute value signs:

$$|3x - 6| < 1 \text{ means the same thing as } -1 < 3x - 6 < 1$$

Epsilon-Delta

Ed: Now divide all terms by 3,

$$-1 < 3x - 6 < 1$$

is equivalent to

$$-\frac{1}{3} < x - 2 < \frac{1}{3}$$

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Now convert back to absolute value (sorry):

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means the same thing as

$$|x - 2| < \frac{1}{3}$$

Epsilon-Delta

Tom: So what’s delta?

Epsilon-Delta

Tom: So what's delta?

Ed: It's $1/3$. We just showed that saying

$$|3x + 1 - 7| < 1$$

means the same thing as saying

$$|x - 2| < \frac{1}{3}$$

so we can say that

$$|f(x) - L| = |3x + 1 - 7| < 1$$

whenever

$$|x - a| = |x - 2| < \frac{1}{3}$$

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Tom: So what does that prove?

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Tom: So what does that prove?

Ed: Only that if $\epsilon = 1$, there is a δ, namely $1/3$, such that

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Tom: Are we done yet?

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Ed: Only that if $\epsilon = 1$, there is a δ, namely $1/3$, such that

$$|3x + 1 - 7| < \epsilon \text{ whenever } |x - 2| < \delta$$

Tom: Are we done yet?

Ed: No, we have to show that no matter what ϵ is given, we can find a δ that makes

$$|3x + 1 - 7| < \epsilon$$

whenever

$$|x - 2| < \delta$$

Epsilon-Delta

Tom: So I have to give you more ϵ values to try?

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Ed: Yup.

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Ed: Yup.

Tom: OK, what if $\epsilon = 1/10$?

Epsilon-Delta

Tom: So I have to give you more ϵ values to try?

Ed: Yup.

Tom: OK, what if $\epsilon = 1/10$?

Ed: Like before, start with the expression $|3x - 6| < 1/10$ and try to manuever it into an equivalent expression that looks like $|x - 2| < something$. That something will be what we call delta.

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Tom: So I have to give you more ϵ values to try?

Ed: Yup.

Tom: OK, what if $\epsilon = 1/10$?

Ed: Like before, start with the expression $|3x-6| < 1/10$ and try to manuever it into an equivalent expression that looks like $|x-2| < something$. That something will be what we call delta.

In the first step we get rid of the absolute value signs:

$$|3x-6| < \frac{1}{10} \text{ means the same thing as } -\frac{1}{10} < 3x-6 < \frac{1}{10}$$

Epsilon-Delta

Ed: Now divide all terms by 3,

$$-\frac{1}{10} < 3x - 6 < \frac{1}{10}$$

is equivalent to

$$-\frac{1}{30} < x - 2 < \frac{1}{30}$$

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Now convert back to absolute value (sorry again):

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Epsilon-Delta

Tom: So δ is $1/30$ this time?

Epsilon-Delta

Tom: So δ is $1/30$ this time?

Ed: Right. We have a smaller ϵ which means $3x + 1$ has to be closer to 7, so we have to take x closer to 2.

$$|3x + 1 - 7| < \frac{1}{10}$$

means the same thing as saying

$$|x - 2| < \frac{1}{30}$$

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Tom: OK, so we can say that

$$|f(x) - L| = |3x + 1 - 7| < \frac{1}{10}$$

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Ed: Right.

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Tom: I’m beginning to see. So are we done yet?

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Tom: Isn't that going to take the rest of our natural lives? We seem to have two ϵ values done and infinity to go. Remember, we only have 24 hours until the bridge opens.

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Tom: What's that?

Ed: We'll come up with a rule for computing a δ. Whatever we are given for ϵ, we apply the rule, and it gives us the value δ we need for this ϵ.

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Tom: And it gives us the value δ we need for this ϵ?

Epsilon-Delta

Ed: Exactly.

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Ed: Exactly.

Tom: For any positive ϵ?

Epsilon-Delta

Ed: Exactly.

Tom: For any positive ϵ?

Ed: Yes. Think of the rule as a function that maps ϵ to a workable δ. As long as the domain of that function is all positive real numbers, we can say there exists a δ for any $\epsilon > 0$ we are given.

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Tom: For any positive ϵ?

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Tom: So now we're done.

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Tom: So now we're done.

Ed: Yes, as soon as we find the rule.

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Tom: So now we're done.

Ed: Yes, as soon as we find the rule.

Tom: Good, I was beginning think we'd never get there.

And how do we find this rule?

Epsilon-Delta

Ed: The same way we found delta values for a specific ϵ, except in place of the given value we just use the symbol ϵ (which conincidentally is my first initial).

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Tom: OK, that means we start with the expression $|3x - 6| < \epsilon$ and try to manuever it into an equivalent expression that looks like $|x - 2| < something$?

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Tom: OK, that means we start with the expression $|3x - 6| < \epsilon$ and try to manuever it into an equivalent expression that looks like $|x - 2| < something$?

Ed: Exactly rignt, and that something will be what we call δ.

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Tom: Can I do the step where we get rid of the absolute value signs?

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Tom: Can I do the step where we get rid of the absolute value signs?

Ed: Sure, why not.

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Ed: Exactly rignt, and that something will be what we call δ.

Tom: Can I do the step where we get rid of the absolute value signs?

Ed: Sure, why not.

Tom: Thanks, that's my favorite part.

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Tom: OK, so this time we say that

$$|3x-6|<\epsilon$$

means the same thing as

$$-\epsilon<3x-6<\epsilon$$

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Tom: OK, so this time we say that

$$|3x - 6| < \epsilon$$

means the same thing as

$$-\epsilon < 3x - 6 < \epsilon$$

Ed: How did that feel?

Epsilon-Delta

Tom: OK, so this time we say that

$$|3x - 6| < \epsilon$$

means the same thing as

$$-\epsilon < 3x - 6 < \epsilon$$

Ed: How did that feel?

Tom: It felt great. Now I'm going to divide each expression by 3 to get something that contains $x - 2$:

$$-\frac{\epsilon}{3} < x - 2 < \frac{\epsilon}{3}$$

Epsilon-Delta

Ed: Good, but now it's time to put back the absolute value signs. I'll spare you and do it myself.

$$-\frac{\epsilon}{3} < x - 2 < \frac{\epsilon}{3}$$

means the same thing as

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means the same thing as

$$|x - 2| < \frac{\epsilon}{3}$$

Tom: Thanks! I see δ should be $\epsilon/3$, so our rule is

$$\delta = \frac{\epsilon}{3}$$

Epsilon-Delta

Ed: Yes, and you can see it will give us a positive δ for any positive ϵ, and because of the way we chose it, δ will have the property that

$$|f(x) - L| = |3x + 1 - 7| < \epsilon$$

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Ed: Yes, for this function. But if you have a different function, we have to do the same thing all over for that function, and we'll end up with a different rule.

Epsilon-Delta

Ed: Yes, and you can see it will give us a positive δ for any positive ϵ, and because of the way we chose it, δ will have the property that

$$|f(x) - L| = |3x + 1 - 7| < \epsilon$$

whenever

$$|x - a| = |x - 2| < \delta$$

Tom: Good. Are we done now?

Ed: Yes, for this function. But if you have a different function, we have to do the same thing all over for that function, and we'll end up with a different rule.

Think of it as job security.

Epsilon-Delta

Tom: Maybe for you, but if my bridge collapses I'll never work again. Are we sure we've eliminated Dari's evil creation?

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Ed: Relax, it's toast. Give me an ϵ less than one.

Epsilon-Delta

Tom: Maybe for you, but if my bridge collapses I'll never work again. Are we sure we've eliminated Dari's evil creation?

Ed: Relax, it's toast. Give me an ϵ less than one.

Tom: Try $1/2$?

Ed: That will do. Now consider this: No matter how small an interval I take around zero, it will always contain some irrational numbers. So I will never be able to say that

$$|f(x) - 0| < \frac{1}{2} \quad \text{whenever} \quad 0 < |x - 0| < \delta$$

no matter what δ is, because every irrational has $|f(x) - 0| = 1$, which is bigger than $1/2$. So, for $\epsilon = 1/2$, there is **no** δ that works, and therefore the limit does not exist.

Epsilon-Delta

Tom: So I can trust limits again? And my bridge isn't going to collapse?

Epsilon-Delta

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Ed: That's right, as long as they satisfy our new definition, which I think should be named after me. How about if we call it the $\epsilon - \delta$ criterion?

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Tom: So I can trust limits again? And my bridge isn't going to collapse?

Ed: That's right, as long as they satisfy our new definition, which I think should be named after me. How about if we call it the $\epsilon - \delta$ criterion?

Tom: It's a deal.