The Derivative

The *derivative* of a function f at $x = a$ is given by

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Both formulas will produce the same result, but one or the other may be easier for a given function.

Example 1

Suppose $f(x) = 2x + 3$. The first formula for the derivative gives

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad = \quad \lim_{h \to 0} \frac{[2(a+h)+3] - [2a+3]}{h}$$

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so

$$f'(a) = \lim_{h \to 0} 2 \quad = \quad 2$$

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Example 2

Suppose $f(x) = x^2 - 2$. The first formula for the derivative gives

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$$f'(a) = \lim_{x \to a} x + a \quad = \quad 2a$$

Question 1

Suppose

$$f(x) = -5x + 1$$

Find $f'(3)$

1. -5x
2. -5
3. -1
4. 5
5. 5x
6. none of the above

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1.	-5x	4.	5
2.	-5	5.	5x
3.	-1	6.	none of the above

2. $f'(3) = -5$

Question 2

Suppose

$$f(x) = x^2$$

Find $f'(x)$

1. -2x
2. -2
3. 2x
4. 2
5. 5x
6. none of the above

Question 2

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Find $f'(x)$

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2. -2
3. 2x
4. 2
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6. none of the above

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The Tangent Line

The **tangent line** to the graph of $f(x)$ at $x = a$ is the line passing through the point $(a, f(a))$ with slope $f'(a)$.

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in the form $y = mx + b$ this becomes

$$y = f'(a)x + f(a) - af'(a)$$

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so after simplifying

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The equation of the tangent line is:

$$y - f(a) = f'(a)(x - a) \quad \text{or} \quad y - 1 = 2(x - 1)$$

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$$y = 2x - 1$$

Question 3

Find the equation of the line tangent to

$$f(x) = x^2 - 4$$

at $x = 1$

1. $y = 2x - 5$
2. $y = 2x + 5$
3. $y = -2x - 5$
4. $y = -2x + 5$
5. $y = -2x$
6. none of the above

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Question 4

Find the equation of the line tangent to

$$f(x) = 2x^2 + 1$$

at $x = -1$

1. $y = 4x - 1$
2. $y = 4x + 1$
3. $y = -4x - 1$
4. $y = -4x + 1$
5. $y = -4x$
6. none of the above

Question 4

Find the equation of the line tangent to

$$f(x) = 2x^2 + 1$$

at $x = -1$

1. $y = 4x - 1$
2. $y = 4x + 1$
3. $y = -4x - 1$
4. $y = -4x + 1$
5. $y = -4x$
6. none of the above

3. $y = -4x - 1$

Question 5

Find the equation of the line tangent to

$$f(x) = \frac{1}{x}$$

at $x = 1$

1. $y = -x - 2$
2. $y = x + 2$
3. $y = -x - 2$
4. $y = -x + 2$
5. $y = -x$
6. none of the above

Question 5

Find the equation of the line tangent to

$$f(x) = \frac{1}{x}$$

at $x = 1$

1. $y = -x - 2$
2. $y = x + 2$
3. $y = -x - 2$
4. $y = -x + 2$
5. $y = -x$
6. none of the above

4. $y = - - x + 2$