Limits

We defined two-sided, left, and right-hand limits:

$$\lim_{x \to a} f(x) = L \qquad \lim_{x \to a-} f(x) = L \quad \text{and} \quad \lim_{x \to a+} f(x) = L$$

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Our definition said nothing about the existence of $f(a)$, or its relation to L if it does exist.

We now define a stronger condition called **continuity** in which we require that $f(a)$ exists and equals L.

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$$\lim_{x \to a^+} f(x) = f(a)$$

Similarly, we say that f is **contiuous from the left** a if $f(a)$ exists and

$$\lim_{x \to a^-} f(x) = f(a)$$

Example 1

Let

$$f(x) = \begin{cases} x^2 & \text{if} \quad x \leq 1 \\ \ln x & \text{if} \quad x > 1 \end{cases}$$

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$$f(x) = \begin{cases} x^2 & \text{if} \quad x \leq 1 \\ \ln x & \text{if} \quad x > 1 \end{cases}$$

At $x = 1$, f has a jump discontinuity and is continuous from the left.

This type of discontinuity cannot be removed.

Example 2

Let

$$f(x) = \frac{x^2 - 2}{x + 2}$$

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$$f(x) = \frac{(x+2)(x-2)}{x+2} \quad = \quad x - 2 \text{ everywhere but } x = -2$$

f has a removable discontintuity at $x = -2$.

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f has a removable discontintuity at $x = -2$.

Define $f(-2) = -4$ to remove it. Then f is continuous everywhere.

Example 3

Let

$$f(x) = \frac{1}{x^2}$$

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Question 1

Suppose

$$f(x) = \begin{cases} \sin x & \text{if} \quad x < 0 \\ e^x & \text{if} \quad x \geq 0 \end{cases}$$

At $x = 0$, f is:

1. continuous from the left
2. continuous from the right
3. continuous
4. undefined
5. none of the above
6. 1,2, and 3

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2. f is continuous from the right (and has a jump discontinuity).

Question 2

Suppose

$$f(x) = \begin{cases} \cos x & \text{if} \quad x < 0 \\ e^x & \text{if} \quad x \geq 0 \end{cases}$$

At $x = 0$, f is:

1. continuous from the left
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6. 1,2, and 3

6. f is continuous so it is continuous from the right and left.

Question 3

Suppose

$$f(x) = \begin{cases} \frac{x^2+2x-3}{x+3} & \text{if} \quad x \neq -3 \\ -2 & \text{if} \quad x = -3 \end{cases}$$

At $x = -3$, f is:

1. continuous from the left
2. continuous from the right
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Suppose

$$f(x) = \begin{cases} \frac{x^2+2x-3}{x+3} & \text{if} \quad x \neq -3 \\ -2 & \text{if} \quad x = -3 \end{cases}$$

At $x = -3$, f is:

1. continuous from the left
2. continuous from the right
3. continuous
4. undefined
5. none of the above
6. 1,2, and 3

5. f has a removable discontinuity at $x = -3$.

Question 4

Suppose

$$f(x) = \frac{1}{x^2 - 1}$$

At $x = \pm 1$, f is:

1. continuous from the left
2. continuous from the right
3. continuous
4. undefined
5. none of the above
6. 1,2, and 3

Question 4

Suppose

$$f(x) = \frac{1}{x^2 - 1}$$

At $x = \pm 1$, f is:

1. continuous from the left
2. continuous from the right
3. continuous
4. undefined
5. none of the above
6. 1,2, and 3

5. f has infinite discontinuities at $x = \pm 1$.