Chapter 1 - In Class Problem Set 1

Gene Quinn

Exercise 1

Determine whether the function defined by the rule of assignment

$$f(x) = 2x^4 - x^2$$

is even, odd, or neither.

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Determine whether the function defined by the rule of assignment

$$f(x) = 2x^4 - x^2$$

is even, odd, or neither.

Answer: even

$$
\begin{aligned}
f(-x) &= 2(-x)^4 - (-x)^2 \\
&= 2x^4 - x^2 \\
&= f(x)
\end{aligned}
$$

Exercise 2

Find the domain of the function

$$f(x) = \frac{1}{1 + x^2}$$

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Answer: All real numbers

Exercise 3

Determine whether the function

$$f(x) = \frac{x}{1 + x^2}$$

is even, odd, or neither

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Determine whether the function

$$f(x) = \frac{x}{1 + x^2}$$

is even, odd, or neither

Answer: odd

$$
\begin{aligned}
f(-x) &= \frac{(-x)}{1+(-x)^2} \\
&= \frac{-x}{1+x^2} \\
&= -\frac{x}{1+x^2} \\
&= -f(x)
\end{aligned}
$$

Exercise 4

A rectangle has an area of $16m^2$.

Express the **perimeter** p as a function of the length of one of its sides.

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Express the **perimeter** p as a function of the length of one of its sides.

Answer: Call the sides w (width) and h (height).

We are given that

$$w \cdot h = 16 \quad \Rightarrow \quad w = \frac{16}{h}$$

so

$$p = 2h + 2w = 2h + 2\left(\frac{16}{h}\right) = 2h + \frac{32}{h}$$

Exercise 5

Express the volume v of a cube as a function of the area a of one face of the cube.

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Express the volume v of a cube as a function of the area a of one face of the cube.

Answer: Let s be the length of each edge. Then

$$
\begin{aligned}
v &= s^3 \\
a &= s^2 \\
s &= \sqrt{a} \\
v &= \left(\sqrt{a}\right)^3
\end{aligned}
$$

Exercise 6

Suppose

$$f(x) = 3x^3 - x^2$$

Is f

- a) an even function?
- b) an odd function?
- c) a polynomial?
- d) a rational function?
- e) an algebraic function?

Exercise 6

Suppose

$$f(x) = 3x^3 - x^2$$

Is f

- a) an even function?
- b) an odd function?
- c) a polynomial?
- d) a rational function?
- e) an algebraic function?

Answer: c) and e)

f is a polynomial

f is algebraic

Exercise 7

Suppose

$$f(x) = \cos x$$

Is f

- a) an even function?
- b) an odd function?
- c) a transcendental function?
- d) a rational function?
- e) a trigonometric function?

Exercise 7

Suppose

$$f(x) = \cos x$$

Is f

- a) an even function?
- b) an odd function?
- c) a transcendental function?
- d) a rational function?
- e) a trigonometric function?

Answer: a), c) and e)

f is an even function

f is transcendental

f is trigonometric

Exercise 8

Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x) = x^2 + 1 \; , \quad \frac{f(a+h) - f(a)}{h}$$

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Evaluate the difference quotient for the given function. Simplify your answer.

$$f(x) = x^2 + 1 \ , \quad \frac{f(a+h) - f(a)}{h}$$

Answer:

$$
\begin{aligned}
\frac{f(a+h) - f(a)}{h} &= \frac{[(a+h)^2 + 1] - [a^2 + 1]}{h} \\
&= \frac{[a^2 + 2ah + h^2 + 1] - [a^2 + 1]}{h} \\
&= \frac{a^2 - a^2 + 2ah + 1 - 1}{h} \\
&= \frac{2ah}{h} \\
&= 2a
\end{aligned}
$$

Exercise 9

Find the domain of the function

$$f(x) = \frac{1}{\sqrt{x^2 - 5x}}$$

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Answer:

$$(-\infty, 0) \cup (5, \infty)$$

Exercise 10

Find the domain and sketch the graph of the function

$$f(x) = \frac{|x|}{x^2}$$

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Find the domain and sketch the graph of the function

$$f(x) = \frac{|x|}{x^2}$$

Answer: The domain is all real numbers except 0:

$$(-\infty, 0) \cup (0, \infty)$$

Exercise 11

Suppose

$$f(x) = \frac{x^2 - 3x}{x^2 + 3x - 1}$$

Is f

- a) a polynomial?
- b) a trigonometric function?
- c) a transcendental function?
- d) a rational function?
- e) an algebraic function?

Exercise 11

Suppose

$$f(x) = \frac{x^2 - 3x}{x^2 + 3x - 1}$$

Is f

- a) a polynomial?
- b) a trigonometric function?
- c) a transcendental function?
- d) a rational function?
- e) an algebraic function?

Answer: d) and e)

f is a rational function

f is algebraic

Exercise 12

Suppose

$$f(x) = 3^x$$

Is f

- a) a polynomial?
- b) a power function?
- c) a transcendental function?
- d) an exponential function?
- e) an algebraic function?

Exercise 12

Suppose

$$f(x) = 3^x$$

Is f

- a) a polynomial?
- b) a power function?
- c) a transcendental function?
- d) an exponential function?
- e) an algebraic function?

Answer: c), and d)

f is a transcendental function

f is an exponential function

Exercise 13

Find an expression for a cubic function f with the property that

$$f(0) = 0, \quad f(2) = 0, \quad \text{and} \quad f(-1) = 0$$

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$$f(0) = 0, \quad f(2) = 0, \quad \text{and} \quad f(-1) = 0$$

Answer:

$$
\begin{aligned}
f(x) \;&=\; (x-0)(x-2)(x+1) \\
&=\; x(x-2)(x+1) \\
&=\; x(x^2 - x - 2) \\
&=\; x^3 - x^2 - 2x
\end{aligned}
$$

Exercise 14

What do the slope and intercept of a linear function

$$f(x) = mx + b$$

represent?

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Answer:

The slope m is the change in f per unit change in x

The intercept is the value of f when $x = 0$.

Exercise 15

Find the domain and range and sketch the graph of the function

$$f(x) = \sqrt{16 - x^2}$$

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Find the domain and range and sketch the graph of the function

$$f(x) = \sqrt{16 - x^2}$$

Answer:

The domain is $[-4, 4]$.

The range is $[0, 4]$.

The graph is the top half of a circle of radius 4, centered at the origin.