MA125 Exam 3

Name:

Key (pink)

1) Find the linearization $L(x)$ at $a = 0$ of the function

$$h(x) = \sqrt[3]{1+x}$$

and use it to estimate $\sqrt[3]{1.1}$.

$h'(x) = \frac{1}{3}(1+x)^{-2/3}$ $\quad h'(0) = \frac{1}{3}$

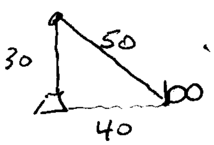
$h(0) = 1$

$L(x) = f(a) + f'(a)(x-a)$

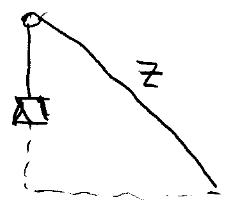
$= 1 + \frac{1}{3}x$

$\sqrt[3]{1+x} = \sqrt[3]{1.1} \Rightarrow x = 0.1$ $\quad \sqrt[3]{1.1} \approx L(0.1) = 1 + \frac{.1}{3} = 1.033\overline{3}$

2) A 2000 lb weight rests on the ground. A cable with no slack runs through a pulley 30 ft directly above the wight to a forklift that is 40 ft from the weight (measured horizontally along the ground). The forklift begins to raise the weight by driving away from it at a constant speed of 2 ft/sec. Assuming the cable does not stretch, how fast is the distance from the ground to the weight increasing when the weight is 15 ft off the ground?



∴ Cable Length $= 80$



$z^2 = x^2 + 30^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt}$

$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$

$15 = h = z - 50$

$\frac{dh}{dt} = \frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt} = \frac{\sqrt{z^2-30^2}}{z}\cdot 2$

$\left.\frac{dh}{dt}\right|_{15} = \frac{\sqrt{65^2-30^2}}{65}\cdot 2$ ft/s

3) Find the derivative of the function

$$h(x) = \tan^{-1}(1+x)$$

$y = \tan^{-1}(1+x) \Rightarrow \tan y = 1 + x$

Taking derivatives: $\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+(1+x)^2} = \frac{1}{2+2x+x^2}$

4) Find the equation of the line tangent to the curve defined implicitly by

$$x^2 - 2xy - y^2 = 2$$

at the point $(3, 1)$.

Taking derivatives,

$$2x - 2xy' - 2y - 2yy' = 0$$

$$2(x-y) - 2y'(x+y) = 0$$

$$2(x-y) = 2y'(x+y)$$

$$y' = \frac{x-y}{x+y} = \frac{3-1}{3+1} = \frac{1}{2}$$

$$(y-1) = \tfrac{1}{2}(x-3) \qquad y = \tfrac{1}{2}x - \tfrac{1}{2}$$

5) Find the derivative of

$$f(x) = \left(3 + \sqrt{2+x}\right)^3$$

$$f'(x) = 3(3+\sqrt{2+x})^2 \frac{d}{dx}(3+\sqrt{2+x})$$

$$= 3(3+\sqrt{2+x})^2 \frac{1}{2\sqrt{2+x}} \cdot \frac{d}{dx}(2+x)$$

$$= 3(3+\sqrt{2+x})^2 \cdot \frac{1}{2\sqrt{2+x}} \cdot 1$$

$$= \frac{3(3+\sqrt{2+x})^2}{2\sqrt{2+x}}$$

6) Find the derivative of

$$g(x) = \ln\left(\frac{x+1}{\sqrt{x-2}}\right)$$

$$g(x) = \ln\left(\frac{x+1}{\sqrt{x-2}}\right) = \ln(x+1) - \tfrac{1}{2}\ln(x-2)$$

$$= \frac{1}{x+1} - \frac{1}{2(x-2)}$$

$$= \frac{x-5}{2(x+1)(x-2)}$$

7) Find the critical numbers and the absolute maximum and minimum of

$$f(x) = \frac{2 + \ln(x)}{x^2} \quad \text{on the interval } [1, 4]$$

$$f'(x) = \frac{x^2\left(\frac{1}{x}\right) - (2+\ln x)(2x)}{x^4} = \frac{x - 4x - 2x\ln x}{x^4}$$

$$= \frac{-x(3+2\ln x)}{x^4} = 0 \text{ if: } x=0 \text{ or } 3+2\ln x = 0 \Rightarrow \ln x = \frac{-3}{2}$$

$$x = e^{-3/2} < 1$$

$\therefore$ no critical values in $[1,4]$

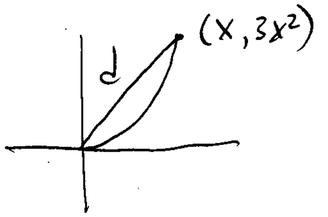
$f(1) = \frac{2}{1} = 2$ (abs. max)

$f(4) = \frac{2+\ln 4}{16}$ (abs min)

8) A particle moves along the curve

$$y = 3x^2$$

As it passes through the point $(1, 3)$, its x-coordinate is increasing at the rate of 2 cm/sec. How fast is the distance from the particle to the origin increasing at this instant?



$$d = \sqrt{(x-0)^2 + (3x^2-0)^2}$$

$$= \sqrt{x^2 + 9x^4}$$

$$\frac{dd}{dt} = \frac{1}{2\sqrt{x^2+9x^4}} \cdot (2x + 36x^3) \cdot \frac{dx}{dt}$$

$$= \frac{38}{2\sqrt{10}} \cdot 2\,\frac{\text{cm}}{\text{sec}} = \frac{38}{\sqrt{10}} \text{ cm/sec}$$

9) Given that

$$y = \ln x$$

a) Find the differential dy

b) Evaluate dy and Δy if $x = 1$ and $dx = \Delta x = 1$.

a) $dy = f'(x)\,dx = \frac{1}{x} \cdot dx$

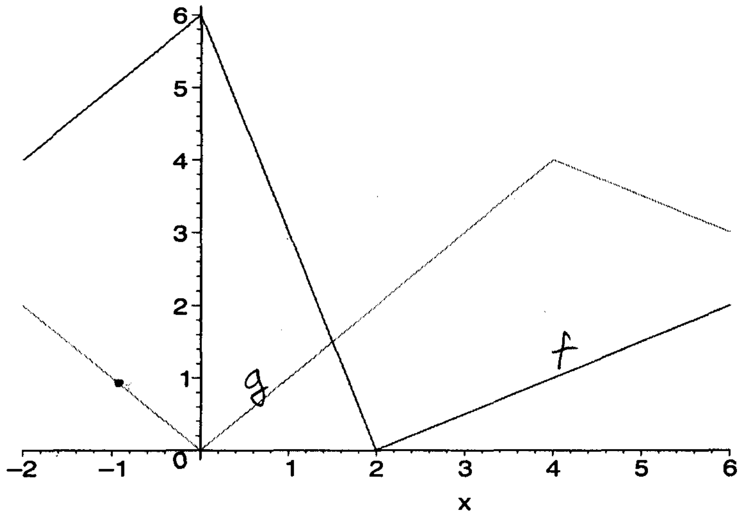
b) $dy = \frac{dx}{x} = \frac{1}{1} = 1$

$$\Delta y = \ln(x + \Delta x) - \ln(x)$$

$$= \ln(1+1) - \ln(1)$$

$$= \ln 2$$

10) The following graph shows two continuous functions $f(x)$ and $g(x)$, on $[-2, 6]$, with $g(0) = 0$



a) Find $(f \circ g)'(-1)$

b) Find $(g \circ f)'(1)$

$$(f \circ g)'(-1) = f'(g(-1)) \cdot g'(-1) \qquad g(-1) = 1$$

$$= f'(1) \cdot (-1) \qquad f'(1) = -3$$

$$= (-3)(-1) = 3$$

$$(g \circ f)'(1) = g'(f(1)) \cdot f'(1) \qquad f(1) = 3$$

$$= g'(3) \cdot (-3) \qquad g'(3) = 1$$

$$= 1(-3) = -3$$