MA125 Exam 3

Name: Key (white)

1) Find the linearization $L(x)$ at $a = 0$ of the function

$$h(x) = \sqrt[4]{1+x}$$

and use it to estimate $\sqrt[4]{1.1}$.

$h'(x) = \frac{1}{4}(1+x)^{-3/4}$ $\qquad$ $h(0) = \sqrt[4]{1} = 1$

$h'(0) = \frac{1}{4}(1^{-3/4}) = \frac{1}{4}$

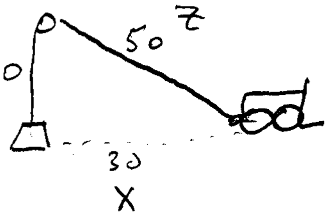
$L(x) = h(a) + h'(a)(x-a)$

$= 1 + \frac{1}{4}x$

$\sqrt{1.1} = \sqrt{1+x} \Rightarrow x = 0.1$ $\qquad$ $\sqrt{1.1} \approx L(0.1) = 1 + \frac{0.1}{4} = 1.025$

2) A 2000 lb weight rests on the ground. A cable with no slack runs through a pulley 40 ft directly above the wight to a forklift that is 30 ft from the weight (measured horizontally along the ground). The forklift begins to raise the weight by driving away from it at a constant speed of 3 ft/sec. Assuming the cable does not stretch, how fast is the distance from the ground to the weight increasing when the weight is 20 ft off the ground?

Initial state:

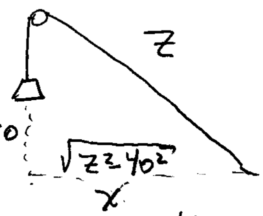


$\therefore$ Cable Length = 90'

$z^2 = x^2 + 40^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt}$

$\frac{dz}{dt} = \frac{x}{z}\frac{dx}{dt}$



$20 = h = z - 50$

$h = 20 \Rightarrow z = 70$

$\frac{dh}{dt} = \frac{dz}{dt}$

$\left.\frac{dh}{dt}\right|_{20} = \frac{\sqrt{70^2 - 40^2}}{70} \cdot 3 \text{ ft/sec}$

3) Find the derivative of

$$h(x) = \tan^{-1}(2+x)$$

$y = \tan^{-1}(2+x)$

$\tan y = 2+x$

taking derivatives, $\sec^2 y \cdot y' = 1$ $\qquad$ $y' = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+(2+x)^2}$

$= \frac{1}{5+4x+x^2}$

4) Find the equation of the line tangent to the curve defined implicitly by

$$x^2 - xy - y^2 = 1$$

at the point $(2, 1)$.

Taking derivatives,

$$2x - y - xy' - 2yy' = 0$$

$$2x - y = y'(x+2y)$$

$$y' = \frac{2x-y}{x+2y} = \frac{2(2)-1}{2+2(1)} = \frac{3}{4}$$

$m = \frac{1}{2}$

$x_1 = 2$

$y_1 = 1$

$y - y_1 = m(x - x_1) \quad y - 1 = \frac{3}{4}(x-2) \quad y = \frac{3}{4}x - \frac{6}{4} + 1 = \frac{3}{4}x - \frac{1}{2}$

5) Find the derivative of

$$f(x) = (1 + \sqrt{1+x})^4$$

$$f'(x) = 4(1+\sqrt{1+x})^3 \cdot \frac{d}{dx}(1+\sqrt{1+x})$$

$$= 4(1+\sqrt{1+x})^3 \cdot \frac{1}{2\sqrt{1+x}} \cdot \frac{d}{dx}(1+x)$$

$$= 4(1+\sqrt{1+x})^3 \cdot \frac{1}{2\sqrt{1+x}} \cdot 1$$

$$= \frac{2(1+\sqrt{1+x})^3}{\sqrt{1+x}}$$

6) Find the derivative of

$$g(x) = \ln\left(\frac{x+1}{\sqrt{x-2}}\right)$$

$$g(x) = \ln\left(\frac{x+1}{\sqrt{x-2}}\right) = \ln(x+1) - \ln(\sqrt{x-2})$$

$$= \ln(x+1) - \frac{1}{2}\ln(x-2)$$

$$= \frac{1}{x+1} - \frac{1}{2}\left(\frac{1}{x-2}\right)$$

$$= \frac{(x-5)}{2(x+1)(x-2)}$$

7) Find the critical numbers and the absolute maximum and minimum of

$$f(x) = \frac{\ln x}{x^2} \quad \text{on the interval } [1,3]$$

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - \ln x\,(2x)}{x^4} = \frac{x(1-2\ln x)}{x^4}$$

$f'(x) = 0$ if $x(1-2\ln x) = 0 \Rightarrow \begin{cases} x = 0 \\ \text{or} \\ \ln x = \frac{1}{2} \Rightarrow x = \sqrt{e} \end{cases}$

$f(1) = 0$

$f(3) = \frac{\ln 3}{9}$

$f(\sqrt{e}) = \frac{1/2}{e} = \frac{1}{2e}$

ABS max: $\frac{1}{2e}$ at $x = \sqrt{e}$

ABS min: 0 at $x = 1$

8) A particle moves along the curve

$$y = 2x^2$$

As it passes through the point $(1,2)$, its x-coordinate is increasing at the rate of 3 cm/sec. How fast is the distance from the particle to the origin increasing at this instant?

$$d = \sqrt{(x-0)^2 + (2x^2-0)^2}$$

$$d = \sqrt{x^2 + 4x^4}$$

$$\frac{dd}{dt} = \frac{1}{2\sqrt{x^2+4x^4}} \cdot (2x + 12x^3) \cdot \frac{dx}{dt}$$

when $x = 1$, $\frac{dd}{dt} = \frac{1}{2\sqrt{5}} \cdot (2+12) \cdot 3 = \frac{21}{\sqrt{5}}$ cm/sec

9) Given that

$$y = \ln(1+x)$$

a) Find the differential dy

b) Evaluate dy and Δy if $x = 0$ and $dx = \Delta x = 1$.

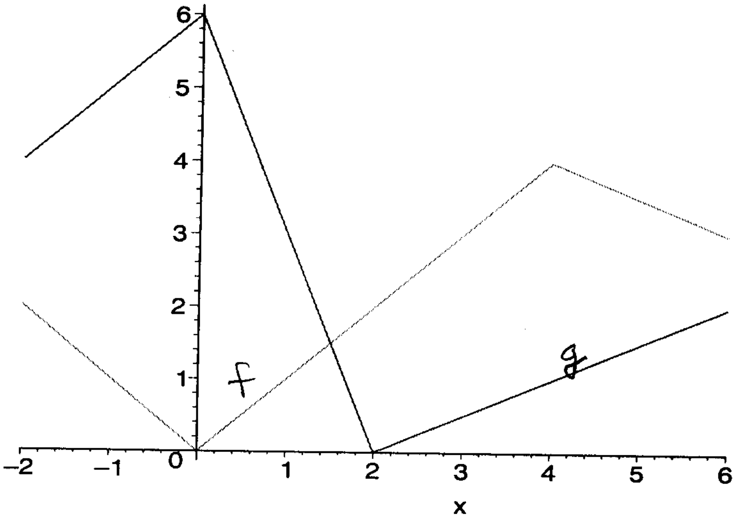
$$dy = y'\,dx = \frac{dx}{1+x}$$

$$dy = \frac{dx}{1+x} = \frac{1}{1+0} = 1$$

$$\Delta y = \ln(1+x+\Delta x) - \ln(1+x) = \ln\left(\frac{1+x+\Delta x}{1+x}\right)$$

$$= \ln\left(\frac{1+1}{1}\right) = \ln 2$$

10) The following graph shows two continuous functions $f(x)$ and $g(x)$, on $[-2, 6]$, with $f(0) = 0$



a) Find $(f \circ g)'(-1)$

b) Find $(g \circ f)'(1)$

$$(f \circ g)'(-1) = f'(g(-1)) \cdot g'(-1)$$
$$= f'(5) \cdot 1$$
$$= (-1/2) \cdot 1 = -1/2$$

$$(g \circ f)'(1) = g'(f(1)) \cdot f'(1)$$
$$= g'(1) \cdot 1$$
$$= -3 \cdot 1 = -3$$