MA125 Quiz 7

Name:

1) (8 pts) Differentiate

$$f(x) = x^2 e^x$$

Product Rule:

$f(x) = g(x) \cdot h(x)$

$g(x) = x^2$

$g'(x) = 2x$

$h(x) = e^x$

$h'(x) = e^x$

$f'(x) = g'h + h'g$

$= 2x e^x + e^x \cdot x^2$

$= xe^x(2+x)$

2) (8 pts) Find the equation of the line tangent to

$$g(x) = \frac{e^x + 1}{x + 1}$$

at the point $(0, 2)$.

$g(x) = \frac{u(x)}{v(x)}$

$g'(x) = \frac{vu' - uv'}{v^2}$

$v(x) = x+1$

$v'(x) = 1$

$u(x) = e^x + 1$

$u'(x) = e^x$

$(v(x))^2 = (x+1)^2$

$g'(x) = \frac{(x+1)e^x - (e^x+1) \cdot 1}{(x+1)^2}$

$= \frac{xe^x + e^x - e^x - 1}{(x+1)^2}$

$= \frac{xe^x - 1}{(x+1)^2}$

(OVER)

The tangent line at $(0,2)$ is

$(y-2) = g'(0)(x-0)$

$g'(0) = \frac{0e^0 - 1}{(0+1)^2} = -1$

$y - 2 = -1(x-0)$

or $y = -x + 2$

3) (9 pts) A parabola $ax^2 + bx + c$ has a tangent line with a slope of 2 at $x = 1$ and a tangent line with a slope of -6 at $x = -1$. If the curve passes through the point (-2,10), find a, b, and c.

Let $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

$$\left.\begin{aligned} f'(1) &= 2a \cdot 1 + b = 2 \\ f'(-1) &= 2a(-1) + b = -6 \end{aligned}\right\} \Rightarrow 2b = -4 \Rightarrow \underline{b = -2}$$

$\Rightarrow 2a - 2 = 2$

$2a = 4$

$\underline{a = 2}$

Now solve for C!

$f(-2) = 2(-2)^2 - 2(-2) + C = 10$

$2(4) + 4 + C = 10$

$12 + C = 10$

$C = -2$

$f(x) = 2x^2 - 2x - 2$