MA125 Quiz 5

Name: KEY

1) (8 pts) Find $f'(x)$ using only the definition of a derivative as a limit if

$$f(x) = 3 - 2x + 4x^2$$

by definition, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

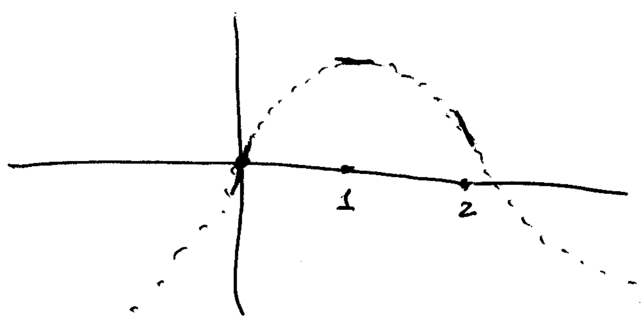
$f(x+h) = 4(x+h)^2 - 2(x+h) + 3 = 4x^2 + 8xh + 4h^2 - 2x - 2h + 3$

$-f(x) = -(4x^2 - 2x + 3)$

$f(x+h) - f(x) = 8xh + 4h^2 - 2h$

$\frac{f(x+h)-f(x)}{h} = \frac{8xh+4h^2-2h}{h} = 8x+4h-2$ if $h \neq 0$. $\lim_{h \to 0} = 8x-2$ by direct subs.

2) (8 pts) Sketch the graph of a function for which $f(0) = 0$, $f'(0) = 3$, $f'(1) = 0$, and $f'(2) = -1$.



many variations are possible.

Correct solutions have:

1) a curve passing through (0,0) with slope of tangent at x=0 of 3

2) A horizontal tangent at x=1

3) A tangent with slope = −1 at x=2

(OVER)

3) (9 pts) Using only the definition of a derivative as a limit, find the derivative of

$$f(x) = x + \sqrt{x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{(x+h) + \sqrt{x+h} - x - \sqrt{x}}{h} = \lim_{h \to 0} \frac{h + \sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} + \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

but $\frac{h}{h} = 1$ if $h \neq 0$, so this is

$$= \lim_{h \to 0} 1 + \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= 1 + \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= 1 + \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = 1 + \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x}}$$

if $h \neq 0$, the h's cancel out leaving

$$f'(x) = 1 + \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = 1 + \frac{1}{2\sqrt{x}}$$

by direct substitution.